NET-BARYON FLUCTUATIONS AT THE QCD CRITICAL POINT

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In the vicinity of the quark-hadron critical point, in the phase diagram of QCD, simple power-law relations constrain the mid-rapidity net-baryon density profile, for different heavy-ion processes, in a unifying scheme. The corresponding scaling variable specifies the proximity of a given experiment to the critical point. An important phenomenological aspect of the critical properties in the baryonic sector is the possibility to observe intermittency effects in the phase space distribution of net baryons.

1 Introduction

Recent investigations concerning the QCD phase diagram $(\rho - T)$ predict the existence of an endpoint along the critical line of the first order quark-hadron phase transition [1]. This critical point, located on a line of nonzero baryonic density $\rho = \rho_c$, is of second order and belongs to the universality class of a $3-d$ Ising system. The QCD critical point communicates with the hadronic world through the fluctuations of a scalar field (σ-field) which carries the quantum numbers of an isoscalar (σ-meson) as the manifestation of a quark condensate, $\sigma \sim \langle \bar{q}q \rangle$, in thermal environment. In the effective theory of the QCD critical point, the classical σ-field is a natural order parameter, the fluctuations of which obey scaling laws dictated by the critical exponents of the $3-d$ Ising system ($\eta \approx 0, \beta \approx \frac{1}{3}, \delta \approx 5, \nu \approx \frac{2}{3}$). In a baryonic environment, however, the chiral condensate is expected to have, at $T = T_c$, a strong dependence on the net-baryon density, driving the σ-field close to zero for $\rho \approx \rho_c$: $\langle \bar{q}q \rangle_\rho \approx \lambda \left(\frac{\rho - \rho_c}{\rho_c} \right) \langle \bar{q}q \rangle_0 + O[(\rho - \rho_c)^2]$ where λ is a dimensionless constant of the order of unity. This dependence suggests a new order parameter, $m = \rho - \rho_c$, associated with the critical properties of the baryonic fluid created in a quark-hadron phase transition. In fact, approaching the critical point in the phase diagram, both the σ-field fluctuations and the fluctuations of the order parameter $m(\vec{x})$ obey the same scaling laws ($\langle \bar{q}q \rangle_\rho \sim m(\vec{x})$).

In the next section we will exploit the scaling properties of the order parameter $m(\vec{x})$, properly adjusted to measurable quantities, in heavy ion collisions. For this purpose we consider net-baryon production in collisions of heavy nuclei with total number of participants A_t and initial energy corresponding to a total size in rapidity $\Delta y = L$.

2   Scaling of the baryonic density

The created baryons in the process of quark-hadron phase transition occupy a cylindrical volume with transverse radius $R_\perp \sim A_\perp^{1/3}$ and longitudinal size L (in rapidity). The parameter $A_\perp$ specifies the effective number of participants, contributing to the transverse geometry of the collision, and it is assumed $A_\perp \approx \frac{A_t}{2}$, valid both for central ($A_\perp = A_{min}$) and non-central collisions. Projecting out the net-baryon system onto the longitudinal direction we end up with a $1 - d$ liquid confined in a finite rapidity region of size L with local density $\rho(y) = \frac{n_b(y)}{\pi R_\perp^2 \tau_f}$, directly related to the measurable net-baryon density in rapidity $n_b(y) = \frac{dN_b}{dy}$. Putting $R_\perp = R_o A_\perp^{1/3}$, we introduce a characteristic volume $V_o = \pi R_o^2 \tau_f$ in terms of the freeze-out time scale $\tau_f \geq 6-8\ fm$ and the nuclear-size scale R_o which contains also any growth effects near the critical point ($R_o \geq 1.2\ fm$).). Using V_o^{-1} as a scale for baryonic, freeze-out densities, the order parameter $m(y)$ of the $1 - d$ baryonic liquid is written:

$$m(y) = A_\perp^{-2/3} n_b(y) - \rho_c \qquad , \qquad 0 \leq y \leq L \tag{1}$$

Near the critical point ($T \to T_c$) the order parameter $m(y)$ obeys a scaling law:

$$m(y) = t^\beta \left[F_o(y/L) + t F_1(y/L) + ... \right] \tag{2}$$

where $t \equiv \frac{T_c - T}{T_c}$ ($T \leq T_c$) and β is the appropriate critical exponent ($\beta \approx 1/3$). The leading term $F_o(y/L)$ in the series (2) is a universal scaling function whereas the nonleading terms $F_i(y/L)$ ($i = 1, 2, ..$) are nonuniversal but A-independent quantities. Using eqs.(1) and (2) we may specify the freeze-out line in the phase diagram by considering the measurable, bulk density at midrapidity $n_b = n_b(L/2)$ as a function of the freeze-out temperature T_f:

$$A_\perp^{-2/3} n_b = \rho_c + t_f^\beta \left[F_o + t_f F_1 + ... \right] \tag{3}$$

where $t_f = \frac{T_c - T_f}{T_c}$ and $F_i \equiv F_i(1/2)$. Integrating now eq.(2) in the interval $0 \leq y \leq L$ we obtain at $T = T_f$:

$$A_\perp^{-2/3} A_t L^{-1} = \rho_c + t_f^\beta (I_o + t_f I_1 + ...) \tag{4}$$

where $I_i = \int_0^1 F_i(\xi) d\xi$. Introducing the variable $z_c = A_\perp^{-2/3} A_t L^{-1}$ we find from eqs.(3) and (4) a scaling law for the net-baryon density at midrapidity:

$$A_\perp^{-2/3} n_b = \Psi(z_c, \rho_c) \qquad ; \qquad z_c \geq \rho_c \tag{5}$$

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where the scaling function $\Psi(z_c, \rho_c)$ has the property $\Psi(z_c = \rho_c, \rho_c) = \rho_c$. In the crossover regime $z_c < \rho_c$, where critical fluctuations disappear, the local density at midrapidity is, to a good approximation, $n_b \approx A_t L^{-1}$ suggesting a continuous extension of the scaling law (5) in this region with $\Psi(z_c, \rho_c) = z_c$ $(z_c < \rho_c)$. It is of interest to note that although the scaling function $\Psi(z_c, \rho_c)$ is continuous at the critical point $(z_c = \rho_c)$, the first derivative is expected to be discontinuous at this point in accordance with the nature of the phase transition (critical point of second order). Keeping only the next to leading term $F_1(y/L)$ in (2) we finally obtain:

$$A_\perp^{-2/3} n_b = \rho_c + \frac{F_o}{F_1}[f(z_c, \rho_c)]^\beta +$$

$$+ C[f(z_c, \rho_c)]^{\beta+1} \tag{6-a}$$

$$A_\perp^{-2/3} n_b = \rho_c + F_o t_f^\beta (1 + t_f \frac{C I_o^{1+\frac{1}{\beta}}}{F_o}) \tag{6-b}$$

$$f(z_c, \rho_c) = \frac{1}{G}(-1 + \sqrt{1 + 2G(z_c - \rho_c)^{1/\beta}});$$

$$z_c \geq \rho_c \tag{6-c}$$

where $C = \frac{F_1}{I_o^{1+\frac{1}{\beta}}}$, $G = \frac{2I_1}{\beta I_o^{1+\frac{1}{\beta}}}$.

The component $F_o(y/L)$ which dominates the order parameter $m(y)$ in the limit $T \to T_c$ is approximately constant in the central region $(y \approx \frac{L}{2}, L \gg 1)$, far from the walls (at the points $y = 0, L$) due to the approximate translational invariance of the finite system in this region. On the other hand, approaching the walls, $F_o(y/L)$ describes the density correlation with the endpoints and obeys appropriate power laws. The solution which fulfils these requirements is:

$$F_o(\xi) = g[\xi(1-\xi)]^{-\beta/\nu} \quad (0 \leq \xi \leq 1) \tag{7}$$

and the final form of the scaling function $\Psi(z_c, \rho_c)$ becomes:

$$\Psi(z_c, \rho_c) = \rho_c + \frac{4^{\beta/\nu}}{B(1-\frac{\beta}{\nu}, 1-\frac{\beta}{\nu})}[f(z_c, \rho_c)]^\beta$$

$$+ C[f(z_c, \rho_c)]^{\beta+1} \; ; \; z_c \geq \rho_c \tag{8-a}$$

$$\Psi(z_c, \rho_c) = z_c \; ; \; z_c < \rho_c \tag{8-b}$$

where $f(z_c, \rho_c)$ is given by eq.(6-c). It is straightforward to show that the discontinuity of the first derivative of $\Psi(z_c, \rho_c)$ at $z_c = \rho_c$ is a nonzero universal constant: $disc\left(\frac{d\Psi}{dz_c}\right)_{\rho_c} \approx 1 - \frac{2}{\pi}$ $(\frac{\beta}{\nu} \approx \frac{1}{2})$ as expected from the characteristic properties of a second-order phase transition.

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3 Approaching the critical point in experiments with heavy ions

Combining eqs.(5-8) one may propose a framework for the treatment of certain phenomenological aspects of the QCD critical point. The scaling law (5) combined with eq.(8) involves three nonuniversal parameters: (a) the critical density ρ_c and (b) the constants C, G which give a measure of the nonleading effects, allowing to accomodate in the scaling function (8) processes not very close to the critical point. We have used measurements at the SPS in order to fix these parameters on the basis of eqs.(5) and (8). More specifically, in a series of experiments (Pb+Pb, S+Au, S+Ag, S+S) with central and noncentral (Pb+Pb) collisions at the SPS net baryons have been measured at midrapidity whereas the scaling variable $z_c = A_\perp^{-2/3} A_t L^{-1}$, associated with these experiments, covers a sufficiently wide range of values ($1 \leq z_c \leq 2$) allowing for a best fit solution. The outcome of the fit is consistent with the choice $G \approx 0$ and the equations (6) are simplified as follows:

$$A_\perp^{-2/3} n_b = \rho_c + \frac{2}{\pi}(z_c - \rho_c) + C(z_c - \rho_c)^4 \tag{9-a}$$

$$A_\perp^{-2/3} n_b = \rho_c + \frac{2I_o}{\pi} t_f^{1/3}(1 + t_f \frac{\pi C I_o^{1/3}}{2}) \tag{9-b}$$

$$A_\perp^{-2/3} n_b(y) = \rho_c + \frac{(z_c - \rho_c)L}{\pi\sqrt{y(L-y)}}$$
$$+ O[(z_c - \rho_c)^4] \tag{9-c}$$

In eqs.(9) we have used the approximate values of the critical exponents $\beta \approx \frac{1}{3}$, $\frac{\beta}{\nu} \approx \frac{1}{2}$ in the $3-d$ Ising universality class. We have also added eq.(9-c) which gives the universal behaviour of the net-baryon density $n_b(y)$, in the vicinity of the critical point ($z_c - \rho_c \ll 1$). The fitted values of the parameters in eq.(9-b) are $\rho_c = 0.81$, $C = 0.68$ and the overall behaviour of the solution is shown in Fig. 1. In turns out that the critical density is rather small, compared to the normal nuclear density $\rho_o \approx 0.17 \ fm^{-3}$: $\rho_c \leq \frac{\rho_o}{5}$ ($R_o \geq 1.2 \ fm$, $\tau_f \geq 6 \ fm$), suggesting that the critical temperature remains close to the value $T_c \approx 140 \ MeV$ obtained in studies of QCD on the lattice at zero chemical potential [4].

The difference $d_c = |z_c - \rho_c|$ in Fig. 1 is a measure of the proximity of a given experiment (A_t, L) to the critical point. We observe that the central Pb+Pb collisions at the SPS ($d_c \approx 1.1$) drive the system into the most distant freeze-out area, from the critical point, as compared to other processes at the same energy. In fact, the most suitable experiments to bring quark matter close to the critical point at the SPS are: S+S ($d_c \approx 0.18$), S_i+S_i ($d_c \approx 0.20$) and C+C ($d_c \approx 0.06$)), central collisions.

4 Critical fluctuations in the baryonic sector

Once the deconfined phase of quark matter has reached the critical point in a particular class of experiments, as discussed in the previous section, strong critical fluctuations are expected to form intermittency patterns both in the pion and net-baryon sector. As already mentioned in the introduction, the origin of these fluctuations can be traced in the presence, at $T = T_c$, of a zero mass field with a classical profile (σ-field) which, under the assumption of a phase transition in local thermal equilibrium, is described by an effective action in $3 - d$, the projection of which onto rapidity space is written as follows [3]:

$$\Gamma_c \approx \frac{\pi R_\perp^2}{C_A} \int_{\delta y} dy \left[\frac{1}{2} \left(\frac{\partial \sigma}{\partial y} \right)^2 + 2 C_A^2 \beta_c^4 \sigma^{\delta+1} \right] \quad ; \quad C_A = \frac{\tau_c}{\beta_c}, \quad \beta_c = T_c^{-1} \quad (10)$$

Equation (10) gives the free energy of the σ-field within a cluster of size δy in rapidity and $R_\perp$ in transverse space. The critical fluctuations generated by (12) in the pion sector have been studied extensively in our previous work [3], therefore, in what follows, we are going to discuss the fluctuations induced by the σ-field in the net-baryon sector, noting that a direct measurement of these fluctuations becomes feasible in current and future heavy-ion experiments. For this purpose we introduce in eq.(10) the order parameter $m(y)$ through the following equations:

$$\sigma(y) \approx F \beta_c^2 m(y) \quad ; \quad F \equiv -\frac{\lambda \langle \bar{q} q \rangle_o}{\rho_c} \quad ; \quad \langle \bar{q} q \rangle_o \approx -3 \ fm^{-3}$$

$$\Gamma_c \approx g_1 \int_{\delta y} dy \left[\frac{1}{2} \left(\frac{\partial \hat{m}}{\partial y} \right)^2 + g_2 |\hat{m}|^{\delta+1} \right] \quad ; \quad \hat{m}(y) = \beta_c^3 m(y) \quad (11)$$

where: $g_1 = F^2 \left(\frac{\pi R_\perp}{C_A \beta_c^2} \right), g_2 = 2 C_A^2 F^4$. The partition function $Z = \int \mathcal{D}[\hat{m}] e^{-\Gamma_c[\hat{m}]}$ for each cluster is saturated by instanton-like configurations [5] which for $\delta y \leq \delta_c$ lead to self-similar structures, characterized by a pair-correlation function of the form:

$$\langle \hat{m}(y) \hat{m}(0) \rangle \approx \frac{5}{6} \frac{\Gamma(1/3)}{\Gamma(1/6)} \left(\frac{\pi R_\perp^2 C_A}{\beta_c^2} \right) F^{-1} y^{-\frac{1}{\delta+1}} \quad (12)$$

The size, in rapidity, of these fractal clusters is $\delta_c \approx \left(\frac{\pi R_\perp^2}{16 \beta_c^2 C_A^2} \right)^{2/3}$ according to the geometrical description of the critical systems [5]. Integrating eq.(12) we find the fluctuation $\langle \delta n_b \rangle$ of the net-baryon multiplicity with respect to the

critical occupation number within each cluster, as follows:

$$\langle \delta n_b \rangle \approx F^{-1} \left(\frac{\pi R_\perp^2 C_A}{2\beta_c^2} \right) \frac{2^{2/3} \Gamma(1/3)}{\Gamma(1/6)} \delta_c^{5/6} \qquad (13)$$

The dimensionless parameter F is of the order 10^2 and the size δ_c, on general grounds ($R_\perp \overset{<}{\sim} 2\tau_c$) is of the order of one ($\delta_c \overset{<}{\sim} 1$). As a result the global baryonic system (in RHIC the size of the system is $L \approx 11$) develops fluctuations at all scales in rapidity since the direct correlation (12) propagates along the entire system through the cooperation of many self-similar clusters of relatively small size ($\delta_c \approx 0.35$ and $\langle \delta n_b \rangle \approx 140$). We have quantified this mechanism in a Monte-Carlo simulation for the conditions of the experiments at RHIC ($L \approx 11$) in order to generate baryons with critical fluctuations. The distribution of these "critical" baryons in the rapidity space for a typical event as well as the corresponding intermittency analysis in terms of factorial moments are presented in Fig. 2.

The intermittency exponent of the second moment F_2 in rapidity is found to be $s_2 \approx 0.18$ which is very close to the theoretically expected value ($\frac{1}{6}$) of a monofractal $1 - d$ set with fractal dimension $\frac{5}{6}$. In conclusion, we have shown that measurements of net-baryon spectra in rapidity provide a valuable set of obsevables in heavy ion experiments, in connection with the phenomenology of the QCD critical point.

References

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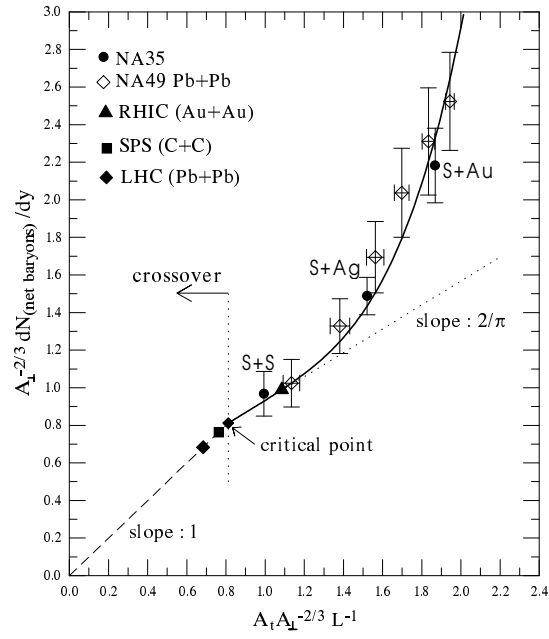
Figure 1: The scaling law (8) is illustrated together with measurements at the SPS. The critical point and the corresponding break in the slope of $\Psi(z_c, \rho_c)$ are also shown.
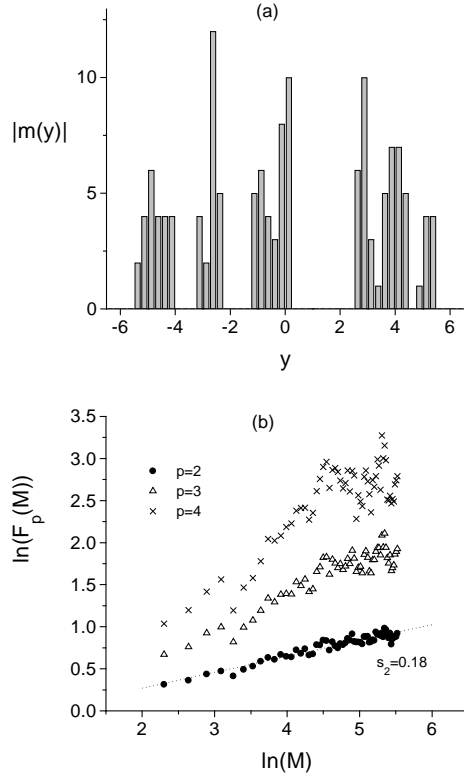
Figure 2: (a) The fluctuations of critical baryons in rapidity for a MC-generated event. (b) The first three factorial moments for the event shown in (a) in a log-log plot. A linear fit determining the slope s_2 (≈ 0.18) of the second moment is also shown.